1.3 Notes and Examples

Name: Block: Seat:

Finding Limits Analytically

Some Basic Limits:
If L, M, c, and k are real numbers (i.e. $\in \mathbb{R}$), with $\lim_{x\to c} f(x) = L$ and $\lim_{x\to c} g(x) = M$

1. $\lim_{x\to c} k =$
2. $\lim_{x\to c} x =$
3. $\lim_{x\to c} x^k =$

1. Examples: Find the following limits
   (a) $\lim_{x\to 2} 5 =$
   (b) $\lim_{x\to -7} x =$
   (c) $\lim_{x\to 3} x^2 =$

Properties of Limits:
If L, M, c, and k are real numbers (i.e. $\in \mathbb{R}$), with $\lim_{x\to c} f(x) = L$ and $\lim_{x\to c} g(x) = M$

1. Sum Rule: $\lim_{x\to c} (f(x) + g(x)) =$
2. Difference Rule: $\lim_{x\to c} (f(x) - g(x)) =$
3. Product Rule: $\lim_{x\to c} (f(x) \cdot g(x)) =$
4. Quotient Rule (if $M \neq 0$): $\lim_{x\to c} \left(\frac{f(x)}{g(x)}\right) =$
5. Constant Multiple Rule: $\lim_{x\to c} (k \cdot f(x)) =$
6. Power Rule (if $L^{a/b} \in \mathbb{R}$, and a, b are integers (i.e. $\in \mathbb{Z}$): $\lim_{x\to c} \left(f(x)^{a/b}\right) =$
7. Composite Function Rule: $\lim_{x\to c} (f(x) \circ g(x)) = \lim_{x\to c} f(g(x)) = f\left(\lim_{x\to c} g(x)\right) =$

2. Examples: Given that $\lim_{x\to a} f(x) = 2$ and $\lim_{x\to a} g(x) = 3$, find the following limits.
   (a) $\lim_{x\to a} 5g(x) =$
   (b) $\lim_{x\to a} \frac{6 + f(x)}{g(x)} =$
   (c) $\lim_{x\to a} [g(x)]^3 =$
   (d) $\lim_{x\to a} f(g(x)) =$
   (e) $\lim_{x\to a} g(x)^{3/2} =$

`http://webspace.ship.edu/msrenault/GeoGebraCalculus/limit_laws.html`

3. Find the following limits

(a) $\lim_{x\to 0} \sqrt{x^2+4} =$

(b) $\lim_{x\to 3} \sqrt[3]{2x^2-10} =$

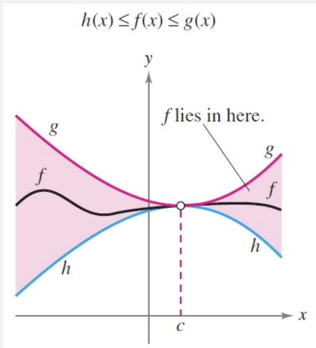
(c) $\lim_{x\to \pi} \cos(x) =$

(d) $\lim_{x\to \frac{\pi}{4}} \sin^2 x =$

Squeeze Theorem: If $h(x) \le f(x) \le g(x)$ for all x in an open interval that contains c, and :

$$\lim_{x\to c} h(x) = \lim_{x\to c} g(x) = L \text{ then } \lim_{x\to c} f(x) = \_\_\_\_\_$$

Theorem: $\lim_{x\to 0} \frac{\sin x}{x} = \_\_\_\_\_$



Theorem: $\lim_{x\to 0} \frac{1-\cos x}{x} = \_\_\_\_\_$

Strategy

Start with Direct Substitution, to determine if the limit is:

Type I. you get a number, you are done.

Type II. your get a number divided by zero, either $\pm\infty$ or DNE (more of this next time)

Type III. you get $\dfrac{0}{0}$ or $\pm\dfrac{\infty}{\infty}$ (indeterminate form), Try:

(a) Factor, divide out, or separate into fractions
(b) Multiply or divide top and bottom by the highest power of x
(c) Rationalize (multiply top and bottom by the conjugate)
(d) Try to make it in the form of $\lim\limits_{x\to 0}\dfrac{\sin x}{x} = 1$ or $\lim\limits_{x\to 0}\dfrac{1-\cos x}{x} = 0$

4. Examples of Type I
   (a) $\lim\limits_{x\to 1}\dfrac{x^2+x+2}{x+1} =$

   (b) $\lim\limits_{x\to 0}\tan x =$

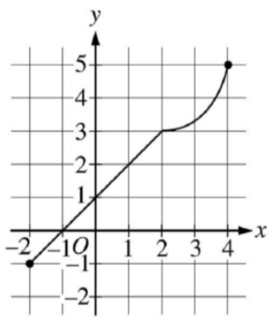
   (c) $\lim\limits_{x\to \pi} x\cos x =$

5. Examples of Type III
   (a) $\lim\limits_{x\to -3}\dfrac{x^2+x-6}{x+3} =$

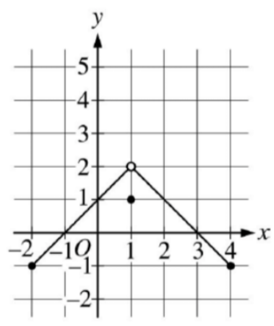
   (b) $\lim\limits_{x\to 0}\dfrac{\sqrt{x+1}-1}{x} =$

   (c) $\lim\limits_{x\to 0}\dfrac{\tan x}{x} =$

More Challenging Examples



Graph of f



Graph of g

6. The graphs of f and g are shown above.
   (a) $\lim_{x\to 1} f(g(x)) =$

   (b) $\lim_{x\to 0} f(f(x)) =$

   (c) If $\lim_{x\to a} g(x) = 1$, find all possible values of a

7. $\lim_{x\to 0} \dfrac{\sin(5x)}{4x}$

8. $\lim_{x\to 0} \dfrac{2x + \sin(x)}{x}$

9. This is a preview of a "one-sided" limit, which is the limit coming from the left:

   $$\lim_{x\to 2^-} \frac{|2-x|}{2-x}$$

10. $\lim_{x\to \frac{\pi}{4}} \dfrac{1 - \tan x}{\sin x - \cos x}$